Variations in offer arrival rates in a stationary search model

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Variations in Offer Arrival Rates in a Stationary Search Model: A Note.

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Abstract: The standard search model predicts that the hazard and the expected accepted wage should be constant over an unemployment spell. This note shows that heterogeneity in job offer arrival rates can generate declining hazards and declining accepted wages, results in conformity with the empirical evidence. In addition, two testable implications are derived. The first is a relationship between the rate of change of the aggregate hazard over time and the variance of the hazard; and the second between the rate of change over time of the expected accepted wage and the covariance between the expected accepted wage and the hazard.

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1 Introduction

The majority of empirical search studies find that the hazard from unemployment and average accepted wages both decline with unemployment spell length (see Devine and Kiefer 1991 for a survey). This finding is in conflict with the standard search model which predicts that both accepted wages and the hazard should be unrelated to unemployment duration (see, e.g., Mortensen 1986 for an exposition of the job search model). The introduction of heterogeneity into the reservation wages of workers (which can arise due to differences in discount rates or search costs), other factors such as the offer arrival rate and the wage distribution kept unchanged, solves only part of the problem. This approach generates a declining hazard from unemployment but also generates an expected accepted wage which *increases* with unemployment duration. In this context, workers with higher reservation wages reject a larger fraction of offered wages, and become employed at a slower rate than workers with lower reservation wages.

The theoretical literature has turned to non-stationary models to explain this observation. Non-stationary models, i.e. models in which factors in the worker's environment are allowed to change over the length of an unemployment spell, can generate declining hazards and declining accepted wages. Examples of non-stationarity in search models include features such as unemployment benefits that decline over time, workers who learn about the wage distribution over time and unemployment spells that have a "scar" effect (see, e.g., Borjas and Heckman 1980; Mortensen 1986; Burdett and Vishwanath 1988). While non-stationarity is certainly a feature of the environment faced by an unemployed worker, other factors may also be important in explaining the empirical observation of declining hazards and declining accepted wages. For instance, many empirical studies have found that variations in offer arrival rates across individuals are an important determinant of variations in the patterns of unemployment (see, e.g., Devine and Kiefer 1991).

This note shows that the broad empirical regularity of declining hazards and declining accepted wages can be accommodated within the framework of a stationary model by allowing job offer arrival rates to vary across otherwise homogeneous workers. The aggregate hazard and the aggregate expected accepted wage are both shown to decline over time. In addition, this model yields two testable implications. First, an equivalence between the rate of change over time of the aggregate hazard and the variance of the aggregate hazard is obtained. Second, a relationship between the rate of change over time of the aggregate expected accepted wage and the covariance between the aggregate hazard and the aggregate expected accepted wage is derived.

2 A Standard Search Model

Consider a search model in which an infinitely-lived unemployed worker seeks to maximize the expected present discounted value of lifetime income. Time is assumed to be continuous and job offers arrive according to a Poisson process with parameter λ. Each job offer is a wage w, taken to be a random draw from a wage offer distribution $F(w)$ with supremum z. While unemployed, the worker receives benefits denoted bdt during time period dt and discounts the future at rate r. Given the stationary environment, the expected present value of unemployment is constant throughout the spell of unemployment, while the expected present value of employment is increasing in the accepted wage. The optimal strategy for a worker is to accept a wage offer only if it exceeds a certain cut-off wage, called the reservation wage. For notational ease, the reservation wage will be denoted by $R(\lambda)$, although it is clear that R is also a function of other parameters. It can be shown (Mortensen, 1986), that $R(\lambda)$ satisfies

$$R(\lambda) = b + \frac{\lambda}{r} \int_{R(\lambda)}^{z} [w - R(\lambda)]\, dF(w).$$

The expected accepted wage, $W(\lambda)$ is simply $E(w|w \geq R(\lambda))$. It can be demonstrated (Mortensen, 1986), that an increase in the offer arrival rate makes workers more discriminating and increases both the reservation wage and the expected accepted wage; i.e. $R'(\lambda) > 0$ and $W'(\lambda) > 0$.

The worker leaves unemployment if a job offer exceeds $R(\lambda)$. Therefore, the hazard from unemployment, denoted $q(\lambda)$, is $\lambda\left[1 - F(R(\lambda))\right]$. The effect of an increase in the job offer arrival rate on the hazard is more complicated, because an increase in λ has a direct positive effect and an indirect negative effect. The second effect arises as an increase in λ increases $R(\lambda)$ which decreases the probability that an offered wage is accepted. The assumption that $F'(w)$ is log-concave is sufficient to guarantee that $q'(\lambda) \geq 0$ (see Van den Berg 1994 for equivalent conditions under which an increase in the job offer arrival rate yields an increase in the hazard).

3 Heterogeneity in Job Offer Arrival Rates

Let there be K groups of workers who are observationally identical except that they receive job offers at different rates[1]. Only unemployment spell lengths and accepted wages are observed by the econometrician who does not know to which group a worker belongs to. All groups of workers become unemployed at time 0. Group i receive offers at rate λ_i, $i = 1, 2, ...K$ and $\lambda_1 > \lambda_2 > .... > \lambda_K$. It is assumed that $F'(w)$, the wage offer density faced by all workers, is log-concave. For group i, the reservation wage, the expected accepted wage and the hazard from unemployment are $R(\lambda_i)$, $W(\lambda_i)$ and $q(\lambda_i)$ respectively.

Since the groups have different arrival rates and therefore exit unemploy-

[1]Situations in which the wage offer distribution faced by workers is the same but the job offer arrival rates they face are different may arise in labour markets where wages are attached to jobs. In such cases, employers have discretion only over the job offers they can make. Anti-discrimination laws may prevent employers from offering lower wages to some workers, but they can choose not to make job offers to workers they do not favour.

ment at different rates, the number of workers belonging to any group i in the unemployed pool changes over time[2]. This is in contrast to the standard search model in which all workers receive offers at the same rate λ. Consequently, in the standard search case, all workers have the same hazard, $q(\lambda)$, and the same expected accepted wage, $W(\lambda)$.

When there is heterogeneity in the offer arrival rate, λ can be treated as a discrete random variable that takes on the value $\lambda_i, i = 1, 2...K$. Interpreting λ as a random variable implies that the reservation wage, the expected accepted wage and the hazard are also random variables since they are functions of λ.

Let $n_i(t)$ be the number of workers of group i who are unemployed at time t. The total number of workers unemployed at time t, $n(t) = \sum_{i=1}^{K} n_i(t)$. Therefore, the proportion of group i workers who are unemployed at time t is $\beta_i(t) = n_i(t)/n(t)$. Notice that $\beta_i(t)$ can also be interpreted as the probability that a worker, drawn randomly from all workers unemployed at time t, belongs to group i. The hazard of workers who are unemployed at time t is a random variable which takes on the value $q(\lambda_i)$ with probability $\beta_i(t)$. The mean of the distribution of the hazard of workers who are unemployed at time t is

$$\bar{q}(t) = \sum_{i=1}^{K} \beta_i(t) q(\lambda_i); \tag{1}$$

$\bar{q}(t)$ can also be interpreted as the aggregate hazard from unemployment at time t. The variance of the hazard at time t is

$$\sum_{i=1}^{K} \beta_i(t) \left[q(\lambda_i) - \bar{q}(t)\right]^2,$$

which simplifies to

$$\sum_{i=1}^{K} \beta_i(t) \left[q(\lambda_i) - \bar{q}(t)\right] q(\lambda_i). \tag{2}$$

Attention is now turned to workers who become employed at time t. The probability that a worker, drawn randomly from all workers who become em-

[2]The empirical implications of the model when there are two groups of workers is explored in Rao Sahib (1998).

ployed at time t, belongs to group i is $\beta_i(t)q(\lambda_i)/\ \overline{q}(t)$. Therefore, the hazard of workers who become employed at time t, is a random variable which takes on the value $q(\lambda_i)$ with probability $\beta_i(t)q(\lambda_i)/\ \overline{q}(t)$. The mean of the distribution of the hazard of workers who become employed at time t is

$$\tilde{q}(t) = \sum_{i=1}^{K} \left[\frac{\beta_i(t)q(\lambda_i)}{\overline{q}(t)} \right] q(\lambda_i).$$

Notice that $\tilde{q}(t)$ can also be interpreted as the aggregate hazard of workers who exit unemployment at time t. Similarly, the expected accepted wage for workers who exit unemployment at time t is a random variable which takes on the value $W(\lambda_i)$ with probability $\beta_i(t)q(\lambda_i)/\overline{q}(t)$. The mean of the distribution of the expected accepted wage at time t is

$$\overline{W}(t) = \sum_{i=1}^{K} \left[\frac{\beta_i(t)q(\lambda_i)}{\overline{q}(t)} \right] W(\lambda_i). \tag{3}$$

$\overline{W}(t)$ can also be interpreted as the aggregate expected accepted wage for workers who exit unemployment at time t.

Since $q'(\lambda) > 0$ and $W'(\lambda) > 0$, workers who belong to groups with higher λ's leave unemployment sooner and have higher accepted wages than workers from groups with lower λ's. Therefore, the covariance between the hazard of the workers who exit unemployment at time t and the expected accepted wage at time t is positive. This is a covariance in an unusual sense, because both the hazard and the expected accepted wage are functions of the same underlying random variable λ. The covariance can be written as:

$$\sum_{i=1}^{K} \left[\frac{\beta_i(t)q(\lambda_i)}{\overline{q}(t)} \right] \left(W(\lambda_i) - \overline{W}(t) \right) \left(q(\lambda_i) - \tilde{q}(t) \right),$$

which simplifies to

$$\sum_{i=1}^{K} \left[\frac{\beta_i(t)q(\lambda_i)}{\overline{q}(t)} \right] \left(W(\lambda_i) - \overline{W}(t) \right) q(\lambda_i). \tag{4}$$

In the next section, the variance and covariance in (2) and (4) will be shown to be related to the rate of change over time of the aggregate hazard and the aggregate expected accepted wage.

4 The Aggregate Hazard and the Aggregate Expected Accepted Wage

First, the aggregate hazard is shown to decline over time. Differentiating the aggregate hazard from (1) w.r.t. t yields

$$\overline{q}'(t) = \sum_{i=1}^{K} \beta_i'(t) q(\lambda_i) \tag{5}$$

where

$$\beta_i'(t) = \frac{n(t)n_i'(t) - n_i(t)n'(t)}{[n(t)]^2}. \tag{6}$$

Since workers leave unemployment at rate $q(\lambda_i)$, the change in $n_i(t)$, denoted $n_i'(t) = -n_i(t)q(\lambda_i)$. It then follows that $n'(t) = -\sum_{i=1}^{K} n_i(t)q(\lambda_i)$. Re-writing (6),

$$\beta_i'(t) = \beta_i(t)\left[\overline{q}(t) - q(\lambda_i)\right]. \tag{7}$$

Notice that equation (7) implies that if a group has a higher than average hazard at time t, then its proportion in the population is declining at time t. Substituting (7) into (5),

$$\overline{q}'(t) = -\sum_{i=1}^{K} \beta_i(t)\left[q(\lambda_i) - \overline{q}(t)\right] q(\lambda_i). \tag{8}$$

Notice that $\overline{q}'(t)$ is the negative of the expression in (2), which is the variance of the hazard of workers who are unemployed at time t; thus, $\overline{q}'(t)$ is negative.

Turning to $\overline{W}(t)$, (3) is differentiated w.r.t. t:

$$\overline{W}'(t) = \left(\frac{\overline{q}(t) \sum_{i=1}^{K} \beta_i'(t) q(\lambda_i) W(\lambda_i) - \overline{q}'(t) \sum_{i=1}^{K} \beta_i(t) q(\lambda_i) W(\lambda_i)}{[\overline{q}(t)]^2} \right).$$

Using (7), (8), substituting $\overline{W}(t)\overline{q}(t)$ for $\sum_{i=1}^{K} \beta_i(t) q(\lambda_i) W(\lambda_i)$, and some straightforward manipulation, yields

$$\overline{W}'(t) = -\sum_{i=1}^{K} \left[\frac{\beta_i(t) q(\lambda_i)}{\overline{q}(t)} \right] \left(W(\lambda_i) - \overline{W}(t) \right) q(\lambda_i).$$

$\overline{W}'(t)$ is the negative of the expression in (4), which is positive. Hence, $\overline{W}(t)$ declines over time.

5 Summary

The standard job search model predicts that the hazard from unemployment and the expected accepted wage should be constant over an unemployment spell. This prediction is at odds with the common empirical observation that the hazard and average accepted wages decline with unemployment duration. It has been shown in this note that a stationary job search model with workers who differ in the rate at which they receive job offers can generate both declining hazards and declining accepted wages.

The model also yields two testable implications. First, an equivalence has been obtained between the rate of change over time of the aggregate hazard and the variance of the hazard. From this perspective, the variance of the hazard in the standard search model is zero. Second, an equivalence between the rate of change over time of the aggregate expected accepted wage and the covariance of the aggregate hazard and the aggregate expected accepted wage is demonstrated.

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